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A holographic perspective on non-relativistic conformal defects

Andreas Karch,^a Piotr Surówka^{a,b} and Ethan G. Thompson^a

^a*Department of Physics, University of Washington,
Seattle, WA 98195-1560 U.S.A.*

^b*Institute of Physics, Jagiellonian University,
Reymonta 4, 30-059 Kraków, Poland*

E-mail: karch@phys.washington.edu, surowka@u.washington.edu,
egthomps@u.washington.edu

ABSTRACT: We study defects in non-relativistic conformal field theories. As in the well-studied case of relativistic conformal defects, we find that a useful tool to organize correlation functions is the defect operator expansion (dOPE). We analyze how the dOPE is implemented in theories with a holographic dual, highlighting some interesting aspects of the operator/state mapping in non-relativistic holography.

KEYWORDS: Gauge-gravity correspondence, Space-Time Symmetries

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1 Introduction

Experimentalists have made great progress in creating fermionic superfluids in the laboratory. An important tool in these experiments is the Feshbach resonance applied to ultracold atomic gases. Using these systems, the BCS superfluid, the unitary Fermi gas, and the Bose-Einstein condensate have all been created [1–3]. However, in all of these situations the fermion pairs form in the s -wave channel rather than the p -wave channel. Experiments have shown that p -wave Feshbach bound states are unstable. It would therefore be of great interest to find a different mechanism for the production of p -wave fermionic superfluids.

Such an alternative mechanism was recently proposed in [4]. This paper presents a model of two interacting fermionic species A and B, where the A-type fermions are confined to live on a two-dimensional defect, while the B-type fermions fill the entire three-dimensional space. The two species interact via the conventional s -wave Feshbach resonance, which induces an attractive interaction among the defect fermions. Because the defect fermions are identical and therefore have identical spin state, this attractive force leads to p -wave pairing. This system may be experimentally realizable using lithium, potassium and optical traps. In [4], several properties of this system are calculated in the weak-coupling limit.

In the unitarity limit, a system of trapped fermionic atoms (such as the one featured in [4]) becomes scale-invariant and enjoys the full Schrödinger symmetry algebra. Such a field theory is referred to as a non-relativistic conformal field theory (NRCFT). Fermions at unitarity are an inherently strongly-coupled system and it is thus difficult to calculate their properties. A solvable toy model of strongly coupled NRCFTs has recently been developed in the form of a holographic description of field theories with the Schrödinger symmetry [5, 6]. In analogy to the familiar AdS/CFT correspondence, these holographic models are tractable on the bulk side when the field theory is strongly-coupled. These models should be useful to study aspects of NRCFTs that are essentially determined by the symmetries and provide a useful laboratory to build intuition about the behavior of these strongly-coupled systems.

Similarly, one would hope that a holographic description of non-relativistic defect CFTs (NRdCFTs) may offer some insight into the models introduced by [4] for p -wave fermionic superfluids. In this paper, we present some initial investigations into holographic descriptions of NRdCFTs. In particular, we will argue that a very useful tool in the analysis of NRdCFTs is the defect operator expansion (dOPE), which is already familiar from the relativistic setting. A holographic description of the dOPE has been carried out for relativistic defect CFTs in [7]. This material is reviewed in section 2.1. We find that many features of relativistic defect CFTs (dCFTs) carry over to the non-relativistic case, with one crucial new feature in the non-relativistic scenario. As shown in section 2.2, the defect Schrödinger geometry allows for the presence of an arbitrary function of the radial coordinate. This function characterizes different NRdCFTs, and is related to the ‘extra’ lightcone dimension present in Schrödinger holography as compared to relativistic holography.

In section 4, a specific example of an NRdCFT is constructed by applying the Null Melvin Twist (NMT) to the Janus solution [8] of type IIB supergravity. This solution represents a particular choice of the arbitrary function mentioned above, and gives rise to a defect theory without matter localized to the defect.

We present possible future directions of this research and our conclusions in section 5 and give more details of our calculations in the appendices.

2 Conformal field theories with defects

2.1 Review of correlators in relativistic dCFTs

Introducing a codimension one defect into a relativistic conformal field theory obviously breaks some of the spacetime symmetries of the theory, such as translation and rotation invariance in the directions transverse to the defect. Conformal defects preserve an $SO(d-1, 2)$ subgroup of the $SO(d, 2)$ conformal symmetry of the d -dimensional CFT that includes scale transformations. Of course, this is just the conformal group in $d-1$ dimensions. Translations, boosts and rotations within the defect act in the natural way on the $d-1$ coordinates along the defect (which we will refer to as \vec{x} and t). What distinguishes a d -dimensional dCFT from a $(d-1)$ -dimensional CFT is that the scale transformations (and also the inversion) also act on the coordinate transverse to the defect (which we will refer to as y). In particular, the dilatation rescales $\vec{x} \rightarrow \lambda\vec{x}$, $t \rightarrow \lambda t$ as well as $y \rightarrow \lambda y$.

Since the symmetry is reduced, correlation functions in a dCFT are much less constrained than correlation functions in a CFT. The constraints have been worked out in full generality in [9]. For example, in a dCFT, scalar operators can have non-trivial one point functions. In a CFT without defect, translation invariance requires the one-point function of any operator other than the identity to vanish. In a dCFT, the dependence on y is not restricted by translation invariance, so scalar operators can have a one-point function $\langle O(t, \vec{x}, y) \rangle = A_O y^{-\Delta}$. Two-point functions of scalar operators O_1 and O_2 of dimension Δ_1 and Δ_2 respectively can even depend on an arbitrary function f

$$\langle O_1(t, \vec{x}, y) O_2(t', \vec{x}', y') \rangle = y^{-\Delta_1} (y')^{-\Delta_2} f(\xi), \tag{2.1}$$

where ξ is the conformally invariant variable

$$\xi = \frac{(x^\mu - (x')^\mu)(x_\mu - (x')_\mu)}{4yy'}. \tag{2.2}$$

A powerful tool in studying dCFTs is the defect operator expansion (dOPE). It allows one to expand any operator $O(t, \vec{x}, y)$ in the full d dimensional dCFT (which, following [7] we'll refer to as an ambient space operator) in terms of defect localized operators $\hat{O}_n(t, \vec{x})$ with scaling dimension $\hat{\Delta}_n$. The $SO(d-1, 2)$ defect conformal group acts on these defect localized operators as the standard conformal group of a $d-1$ dimensional non-defect CFT, so the correlation functions of the \hat{O}_n operators obey the standard non-defect CFT constraints [10]. The constraints on the correlation functions of the full dCFT, like the one-point and two-point functions of scalar operators quoted above, can then be understood as a consequence of the dOPE together with the standard $(d-1)$ -dimensional non-defect CFT constraints on the correlation functions of the \hat{O}_n .

The dOPE of a scalar operator reads [9]

$$O(t, \vec{x}, y) = \sum_n \frac{B_O^{\hat{O}_n}}{y^{\Delta - \hat{\Delta}_n}} \hat{O}_n(t, \vec{x}). \tag{2.3}$$

In terms of the dOPE, the one-point function of the operator O in the full dCFT can be understood as the coefficient B_O^1 of the identity operator in the dOPE, since this is the only operator which can have a non-vanishing expectation value in the $d-1$ dimensional CFT with $\langle 1 \rangle = 1$. Similarly, the free function f appearing in the scalar 2-point function can be expanded as

$$f(\xi) = \sum_{n,m} B_O^{\hat{O}_n} B_O^{\hat{O}_m} y^{\hat{\Delta}_n} (y')^{\hat{\Delta}_m} \langle \hat{O}_n(t, \vec{x}) \hat{O}_m(t, \vec{x}') \rangle. \tag{2.4}$$

The standard constraints on correlation functions in a $d-1$ dimensional CFT¹ ensure that the right hand side indeed only depends on ξ .

¹In a Lorentzian signature CFT one has to be careful about the correct $i\epsilon$ prescription when using the results of [10]. As explained, for example, in [11], for a time-ordered two-point function of O with itself one has

$$\langle T(\hat{O}(t, \vec{x}) \hat{O}(0, 0)) \rangle = \frac{1}{(-t^2 + |\vec{x}|^2 + i\epsilon)^\Delta} \tag{2.5}$$

The simplest example of a dCFT that proved to be useful for studying the dual gravitational description is the “no-brane case” [7]. In a CFT without defect one can promote the line $y = 0$ to a defect and interpret the usual continuity of all fields and their first derivatives as boundary conditions imposed at the defect. In this case the dOPE just becomes a standard Taylor expansion, that is $\hat{O}_n = \partial_y^n O$ with $\hat{\Delta}_n = \Delta + n$.

2.2 The non-relativistic dOPE

The constraints imposed on correlation functions in a $d - 1$ dimensional non-relativistic conformal theory (by which we still mean time plus $d - 2$ spatial dimensions) without defect are quite different from those in a relativistic theory. For example, the two-point function between two scalar operators reads [12, 13]

$$G(t, \vec{x}) = \left\langle T \left(O(t, \vec{x}, y) O^\dagger(0, 0) \right) \right\rangle = Ct^{-\Delta} \exp \left(-iN_0 \frac{|\vec{x}|^2}{2t} \right). \quad (2.7)$$

In this expression, Δ is the dimension of the operator O , defined as in the relativistic case as the eigenvalue under rescalings D ,

$$[D, O(0)] = i\Delta O(0). \quad (2.8)$$

The non-relativistic conformal algebra has a central element N , the particle number. Correspondingly, scalar operators are not just classified by their scale dimension, but in addition one needs to specify their particle number [13]

$$[N, O(0)] = N_O O(0). \quad (2.9)$$

The two-point function depends on both Δ and N_O . In the dual gravitational description this extra quantum number plays a crucial role, as we will demonstrate in the next section.

Correspondingly, correlation functions in an NRdCFT differ significantly from their relativistic counterparts. In [4] it was shown that the two-point function of two defect localized operators $\hat{O}(t, \vec{x})$ takes exactly the same form (2.7), as expected. However, the two-point function of two ambient space operators $O(t, \vec{x}, y)$ can again depend on a free function $f(\xi_{\text{NR}})$ where²

$$\xi_{\text{NR}} = \frac{(t - t')}{yy'}. \quad (2.12)$$

whereas a non-ordered two-point function would be

$$\langle \hat{O}(t, \vec{x}) \hat{O}(0, 0) \rangle = \frac{1}{(-(t - i\epsilon)^2 + |\vec{x}|^2)^\Delta}. \quad (2.6)$$

²To see that ξ_{NR} is invariant under the non-relativistic conformal group note that the most general transformation can be written as [12]

$$t \rightarrow \frac{\alpha t + \beta}{\gamma t + \delta}, \quad \vec{x} \rightarrow \frac{\mathcal{R}\vec{x} + \vec{v}t + \vec{a}}{\gamma t + \delta}, \quad y \rightarrow \frac{y}{\gamma t + \delta} \quad (2.10)$$

where \mathcal{R} is a rotation matrix and $\alpha\beta - \gamma\delta = 1$. Under this transformation one gets

$$(t - t') \rightarrow \frac{(\alpha t + \beta)(\gamma t' + \delta) - (\alpha t' + \beta)(\gamma t + \delta)}{(\gamma t + \delta)(\gamma t' + \delta)} = \frac{t - t'}{(\gamma t + \delta)(\gamma t' + \delta)}. \quad (2.11)$$

Still, the dOPE is just as constrained in the NRdCFT as it was in the relativistic dCFT. Time and space translation invariance along the defect prevent t and \vec{x} from appearing explicitly in the expansion coefficients, which therefore must be purely functions of y . Scale invariance then fixes the NRdOPE to have exactly the same form (2.3) as the relativistic dOPE. As a corollary, the one-point function in the NRdCFT will also be given by the relativistic expression $\langle O(t, \vec{x}, y) \rangle = A_O y^{-\Delta}$. The upshot is that *the reduction of correlation functions of ambient operators to correlation functions of defect localized operators via the dOPE is identical in the NRdCFT and the relativistic dCFT*. The correlators of the defect localized operators are then governed by the corresponding standard expressions for a non-defect CFT (relativistic and non-relativistic respectively) in $d - 1$ spacetime dimensions.

3 The dOPE, relativistic and non-relativistic, as a mode expansion

3.1 Gravitational description of relativistic dCFTs

The AdS/CFT correspondence provides a description of certain d dimensional CFTs in terms of $d + 1$ dimensional gravitational duals. The $SO(d, 2)$ conformal group of the field theory gets mapped to the isometry group of the dual spacetime metric. Up to the curvature radius, which encodes the coupling constant of the CFT, this fixes the dual background uniquely to be AdS_{d+1} . Relativistic dCFTs only have an $SO(d - 1, 2)$ symmetry, so the dual $d + 1$ dimensional metric has more freedom. The $SO(d - 1, 2)$ can be made manifest by foliating the $d + 1$ dimensional spacetime with AdS_d slices,

$$ds^2 = \frac{e^{2A(r)}}{z^2}(-dt^2 + d\vec{x}^2 + dz^2) + dr^2, \tag{3.1}$$

where as in the last section \vec{x} denotes the $(d - 2)$ spatial coordinates along the defect. The warpfactor $A(r)$ is completely undetermined by symmetry considerations. Of course, for any particular dCFT/gravity dual pair, $A(r)$ is determined by solving the equations of motion in the bulk.³ One example is the Janus solution of [8], which is a solution of type IIB supergravity.

Relativistic AdS/CFT pairs come with an operator/field correspondence where every operator on the boundary is dual to a field in the bulk. Let us for now focus on operators that are dual to a scalar fields $\phi_{d+1}(t, \vec{x}, z, r)$ in the $d + 1$ dimensional bulk. Using a separation of variables ansatz,

$$\phi_{d+1}(t, \vec{x}, z, r) = \sum_n \psi_n(r) \phi_{d,n}(t, \vec{x}, z), \tag{3.2}$$

one can decompose the bulk scalar into defect-localized bulk modes $\phi_{d,n}$ (that is, they only depend on the coordinates of the defect and the radial coordinate of the bulk), each of which is an eigenfunction of the AdS_d Laplacian ∇_d^2 acting on the slice coordinates t, \vec{x} and r :

$$\nabla_d^2 \phi_{d,n}(t, \vec{x}, z) = m_n^2 \phi_{d,n}(t, \vec{x}, z). \tag{3.3}$$

³Following [7], we use the standard AdS/CFT terminology and refer to the generic points of the $d + 1$ dimensional spacetime of the gravitational dual as the bulk. In contrast, the spacetime on which the field theory lives is referred to as the defect and its ambient space.

It was shown in [7] that this mode decomposition in the bulk is the dual of the dOPE of the dual dCFT operator on the boundary. While we will mostly focus on free scalar fields, it was argued in [7] that such a decomposition will always be possible, as any bulk field of mass M_0 transforming in some representation of $SO(d, 2)$ will decompose into a tower of AdS_d modes inhabiting representations of the preserved isometry group $SO(d - 1, 2)$. As the modes $\phi_{d,n}(t, \vec{x}, z)$ satisfy a standard scalar wave equation with mass squared m_n^2 , they are naturally dual to the defect localized operators $\hat{O}_n(t, \vec{x})$ appearing in the dOPE (2.3) with the dimensions given by the standard AdS/CFT relation,

$$\hat{\Delta}_n = \frac{d-1}{2} + \frac{1}{2}\sqrt{(d-1)^2 + 4m_n^2}. \tag{3.4}$$

The m_n^2 are the eigenvalues of the radial equation that is obtained from solving the equations of motion for ϕ_{d+1} that follow from the ansatz (3.2). For a free scalar field that is

$$\psi_n''(r) + dA'(r)\psi_n'(r) + e^{-2A(r)}m_n^2\psi_n(r) - M_0^2\psi_n(r) = 0. \tag{3.5}$$

This will receive corrections from various interactions. An important point to note is that already for a free scalar field the spectrum of dimensions $\hat{\Delta}_n$ is not determined from the dimension $\Delta = \frac{d}{2} + \frac{1}{2}\sqrt{d^2 + 4M_0^2}$ of the ambient operator O dual to ϕ_{d+1} alone, but the dimensions also depend on the warpfactor $A(r)$. The coefficient in the dOPE can be extracted from the wavefunctions $\psi_n(r)$ (see [7] for details and an explicit demonstration that this procedure recovers the standard Taylor series in the “no-brane” case of pure AdS_{d+1} foliated by AdS_d slices).

To verify that the mode decomposition indeed reproduces the dOPE one can study the behavior of the full field $\phi_d(t, \vec{x}, z, r)$ in the limit of large r . The component of the boundary of the spacetime that is obtained by going to $r \pm \infty$ is an ambient space point, so asymptotically the spacetime should approach AdS_{d+1} , as locally the ambient space theory is a d dimensional non-defect CFT. In terms of the warpfactor this means that $A(r)$ has to asymptotically approach r , as the metric (3.1) with $e^A = \cosh(r)$ is just AdS_{d+1} . With this choice the asymptotic AdS_{d+1} geometry can be brought into standard flat slicing form

$$ds^2 = \frac{1}{\tilde{z}^2}(-dt^2 + d\vec{x}^2 + dy^2 + d\tilde{z}^2) \tag{3.6}$$

by the change of coordinates

$$y = z \tanh(r) \rightarrow z, \quad \frac{1}{\tilde{z}} = \frac{\cosh(r)}{z} \rightarrow \frac{e^r}{2z} \tag{3.7}$$

The standard AdS/CFT relations give the vacuum expectation value (even in the presence of sources due to the insertions of other operators) of the ambient operator O as the coefficient of \tilde{z}^Δ in $\phi_{d+1}(t, \vec{x}, y, \tilde{z})$ in the limit $\tilde{z} \rightarrow 0$. On the other hand, expanding mode by mode the ϕ_n behave at large r and small z as $\langle \hat{O}_n(t, \vec{x}) \rangle e^{-\Delta r} z^{\hat{\Delta}_n}$. Using the asymptotic change of variables (3.7) one can see that the vacuum expectation values of \hat{O}_n contributes⁴

⁴One subtlety explained in detail in [7] we are glossing over here is that at large r we don't just get contributions from the leading small z behavior of the modes ϕ_d (which is entirely determined by its dimension) but in fact all powers of z contribute. Solving the equations of motion for ϕ_d recursively in a power series in z one can identify these higher powers of z as the contributions of the descendents of the primaries \hat{O}_n to the dOPE.

to the expectation values of the ambient operator O exactly with the coefficient $y^{\hat{\Delta}_n - \Delta}$ as it appears in the dOPE (2.3)

3.2 Gravity dual to non-relativistic CFTs

For a relativistic conformal field theory with gravity dual, the spacetime geometry was essentially fixed to be AdS by the symmetries alone, up to the overall curvature radius. Metrics that exhibit the non-relativistic conformal group (often referred to as the Schrödinger group) as their isometries were analyzed in [5, 6]. It was found that the only way to realize the symmetries of a d -dimensional NRCFT was to have a dual spacetime with *two* extra dimensions.⁵ The corresponding metric, which now is usually referred to as the Schrödinger metric Sch_{d+2} , is

$$ds^2 = -\frac{dt^2}{\tilde{z}^4} + \frac{1}{\tilde{z}^2}(-2dvdt + d\vec{x}^2 + d\tilde{z}^2 + dy^2) \tag{3.8}$$

In this case, there are actually two independent structures in the metric allowed by the symmetries. The $\frac{dt^2}{\tilde{z}^4}$ piece all by itself is symmetric under the full Schrödinger group, as is the rest of the metric (which is just AdS_{d+2} in lightcone coordinates). The constant coefficient of $\frac{dt^2}{\tilde{z}^4}$ can always be scaled to unity by rescaling t and v , but the freedom to have two independent metric structures that respect the full symmetry will be important when we generalize to the defect case. Later it was shown that $Sch_5 \times S^5$ can actually be found as a solution to type IIB supergravity [14–16].

As we pointed out above, one important aspect of this duality is that the gravitational dual actually has two extra dimensions; that is, a d -dimensional NRCFT (again, we uniformly refer to a theory with $d-1$ space plus one time dimension as a d dimensional theory) is dual to Sch_{d+2} . The extra lightlike direction v is needed to compensate the transformations of $d\vec{x}^2 + dy^2$ under Galilean boosts. Typically v is compact and the inverse radius corresponds to the mass of the basic particles described by the NRCFT (and, by convenient choice of units, can be set to unity). The momentum M in the v direction on the gravity side corresponds to the conserved particle number in the field theory. As a consequence, the operator/field dictionary is significantly modified in non-relativistic gauge/gravity duality.

As we mentioned before, operators in an NRCFT are typically taken to have a fixed particle number, see e.g. [13]. To relate this to the more familiar concept of states in non-relativistic quantum mechanics having a fixed particle number, one can note that NRCFTs have an operator/state map that relates the dimension of an operator of particle number $N_0 = M$ to the energy of a state in a harmonic trap with the same particle number [13]. On the gravity side any field $\phi_{d+2}(t, \vec{x}, y, v, \tilde{z})$ can be decomposed into plane waves along the extra v dimension

$$\phi_{d+2}(t, \vec{x}, y, v, \tilde{z}) = e^{i\frac{M}{2\pi}v} \phi_{d+1}(t, \vec{x}, y, \tilde{z}). \tag{3.9}$$

The momentum M maps to particle number in the field theory. This way the operator/field map associates to any given field in the bulk not just one operator, but an infinite tower of

⁵ As in previous sections we will refer to the d spacetime coordinates as t, \vec{x} and y where we singled out one of the $d-1$ spatial coordinates as y since it will play the role of a coordinate transverse to the defect once a defect is introduced.

operators with particle number $M = 0, 1, 2, 3, \dots$. Using the wave equation for a free scalar field⁶ of mass M_0 , one can see that the dimensions of the dual operators are [5, 6]

$$\Delta_M = \frac{d+1}{2} + \sqrt{M_0^2 + M^2 + \frac{(d+1)^2}{4}}. \quad (3.10)$$

In the large M limit this just becomes $\Delta \sim M$. So instead of having a single operator one has a whole tower O_M . The dimensions of the higher O_M are not simple multiples of O_0 . The fact that all the Δ_M as given by (3.10) are determined in terms of M and a single number M_0 is an artifact of using the free wave equations. Higher order terms in the Lagrangian (in particular a coupling of ϕ_{d+2} to the massive vector that is typically used as the matter to support the Schrödinger geometry as a solution to Einstein's equations) will make the dimensions of the various O_M completely independent. As a consequence, once one constructs the gravity dual for a NRdCFT, one should no longer expect the mode decomposition in the bulk to represent a single dOPE but instead it has to yield the dOPE for every O_M ; again one would not expect the different O_M that are dual to a given scalar to have dOPEs that are related in an obvious way. What will be new in the case of a dCFT is that we will see this independence of the dOPEs of different O_M in the same tower of operators dual to the scalar field ϕ_{d+2} already for the simple example of a free scalar field.

3.3 Gravitational description of the non-relativistic dOPE

A d -dimensional NRdCFT preserves a subgroup of the non-relativistic conformal algebra that is equivalent to the non-relativistic conformal algebra in $d - 1$ dimensions. As in the relativistic case, the difference between a d dimensional NRdCFT and a $d - 1$ dimensional NRCFT is that the action of some of the generators (in particular the dilatation operator) in the NRdCFT do not just involve the coordinates on the defect, but also transverse to the defect. When trying to construct a gravitational background that respects the symmetries of a d dimensional NRdCFT in the same spirit as we did in the relativistic case (reviewed in section 3.1), naively one would simply take a $d + 2$ dimensional spacetime and slice it in terms of Sch_{d+1} slices. Just as in eq. (3.1) the Sch_{d+1} on each slice in the full metric is then multiplied by a warpfactor $e^{A(r)}$ that is undetermined by symmetry considerations. However, as we just reviewed in section 3.2, there are actually two independent structures that can appear in the metric consistent with the $d - 1$ dimensional non-relativistic conformal symmetry: $-dt^2/z^4$ and AdS_{d+1} in light cone coordinates (with a lightlike direction compactified). Consequently we have two independent warpfactors in the most general form of a metric that respects the symmetries of a d -dimensional NRdCFT

$$ds^2 = e^{2A(r)} \left(-e^{2B(r)} \frac{dt^2}{z^4} + \frac{-2dvdt + d\vec{x}^2 + dz^2}{z^2} \right) + dr^2. \quad (3.11)$$

As in the relativistic case, we know that far away from the defect the dCFT should recover the full symmetry group of a d dimensional NRCFT. That is, for large r we want the

⁶Note that the only non-trivial components of the inverse metric are $g^{vv} = 1, g^{vt} = g^{tv} = -z^2, g^{xx} = g^{yy} = g^{zz} = z^2$. This way the Laplacian on Sch_{d+2} is essentially equal to the Laplacian on AdS_{d+2} . The only effect of the dt^2 term in the metric is to lead to a shift of the mass term from M_0^2 to $M_0^2 + M^2$.

warped Sch_{d+1} metric in (3.11) to turn into Sch_{d+2} . In order to see what condition this imposes on the warpfactors $A(r)$ and $B(r)$ it is instructive to write Sch_{d+2} in the warped Sch_{d+1} form. Indeed it is easy to see that for

$$e^{A(r)} = e^{B(r)} = \cosh(r) \tag{3.12}$$

the same change of variables (3.7) as in the relativistic case takes the metric in (3.11) into (3.8).⁷ So for a general NRdCFT, asymptotically we have to have $A \sim B \sim r$.

As we saw in section 2.2, the non-relativistic dOPE has exactly the same form as the relativistic dOPE. Given this fact it seems somewhat puzzling that symmetries allow the metric to contain a second free function $B(r)$. If $A(r)$ encodes the dOPE, what information does $B(r)$ carry? In order to shed light on this question, we will once more look at the wave equation for a free scalar of mass M_0 . We make a mode-decomposition ansatz for the field ϕ_{d+2} as in (3.2),

$$\phi_{d+2}(t, \vec{x}, v, z, r) = \sum_n \psi_n(r) \phi_{d+1,n}(t, \vec{x}, v, z). \tag{3.13}$$

Plugging this into the mode equation we again get

$$\nabla_{d+1}^2 \phi_{d+1,n}(t, \vec{x}, v, z) = m_n^2 \phi_{d+1,n}(t, \vec{x}, v, z), \tag{3.14}$$

where this time ∇_{d+1}^2 is the Laplacian on AdS_{d+1} . As in the non-defect case we take the ansatz that $\phi_{d+1}(t, \vec{x}, v, z) = e^{iMv} \phi_d(t, \vec{x}, z)$ to encode the particle number M of the dual operator. The m_n^2 can again be obtained as eigenfunctions of a radial equation which this time reads⁸

$$\psi_n''(r) + (d+1)A'(r)\psi_n'(r) + e^{-2A(r)}m_n^2\psi_n(r) - (M_0^2 + M^2 e^{2B(r)-2A(r)})\psi_n(r) = 0. \tag{3.15}$$

The remaining steps in the derivation of the dOPE from the mode expansion are identical to the relativistic case reviewed in section 3.1, since the asymptotic change of variables (given by (3.7)) in the two cases is identical. Note that for the special case $B = A$ the particle number M simply leads to a shift of the mass squared from M_0^2 to $M_0^2 + M^2$ as in the non-defect case. The mode equation then is basically the same as in the relativistic case; the one difference is that the $A'\psi_n'$ term has a coefficient $d+1$ instead of the d in the relativistic case (as the additional v direction contributes one additional power of e^A to the square root of the determinant of the metric). The non-relativistic d dimensional dCFT knows its origin from a relativistic $d+1$ dimensional dCFT.

⁷To see this note that $\cosh^4(r)dt^2/z^4$ directly turns into dt^2/\tilde{z}^4 under (3.7). The remainder of the metric then is just AdS_{d+1} written in AdS_d slicing and so we know that (3.7) takes this into the standard flat slicing form (3.6) of AdS_{d+1} .

⁸Note that the only non-trivial components of the inverse metric this time are $g^{vv} = e^{2B-2A}$, $g^{vt} = g^{tv} = -z^2 e^{-2A}$, $g^{xx} = g^{zz} = z^2 e^{-2A}$, $g^{rr} = 1$. As a consequence, in an analogy with the non-defect case, the only contribution from the dt^2 term in the metric is a shift of the mass parameter proportional to M^2 , but this time this shift has a non-trivial r -dependence unless $A = B$. What is important for us is that in the separation of variables ansatz this term only affects the radial wave equation and so the $\phi_{d+1,n}$ can again be taken as eigenfunctions of the AdS_{d+1} Laplacian.

More interesting however is the case when $A \neq B$ (of course, asymptotically they have to approach one another, but they can in general be different functions of r). In this case the M -dependent mass shift is multiplied by a free function of r . Hence the eigenvalues m_n^2 and the corresponding eigenfunctions will be highly non-trivial functions of M , because M doesn't simply give an additive shift in the eigenvalue. Thus, we get completely different dOPEs for different values of M . As we emphasized above, this was to be expected. Any field in the NRCFT or NRdCFT setting is not dual to a single operator, but to a whole tower of operators with particle number M . Two free functions are possible in the defect geometry, since we are not just encoding one dOPE, but really an infinite tower of different dOPEs for an infinite tower of ambient space operators O_M .

4 The non-relativistic Janus solution: a specific example of an NRdCFT

In this section, we present an example of the holographic NRdCFTs discussed in previous sections, which we call the non-relativistic Janus solution. The solution is constructed by applying the Null Melvin Twist to the Janus solution of type IIB supergravity.

4.1 The relativistic Janus solution

In [8], an explicit domain-wall solution to the type IIB equations of motion was constructed. The solution includes a non-trivial dilaton, metric and five-form field strength. The geometry is asymptotically AdS, and therefore admits a dual field theory description. The spacetime of the dual field theory is divided into two regions by a defect, and the coupling constant takes a different value in either half of the spacetime. This two-faced nature of the field theory led the authors to call the solution the Janus solution. Janus is non-supersymmetric, and stability is therefore a concern. However, strong evidence for the stability of the solution was presented in [17]. The dual field theory was further investigated in [18], where it was argued that the field theory was a particular deformation of $\mathcal{N} = 4$ SYM, with no matter fields living on the defect. In this paper, our interest in Janus is that it provides an explicit solution to IIB supergravity with a dCFT dual, which we can, in turn, use to generate an explicit solution to IIB supergravity with an NRdCFT dual.

In our conventions, the metric, dilaton and five-form for the Janus solution in the Einstein frame are

$$ds_E^2 = L^2 e^{2A(\mu)} \left(\frac{-d\tau^2 + dx^2 + dy^2 + dz^2}{z^2} + d\mu^2 \right) + L^2 ds_{S^5}^2, \tag{4.1}$$

$$\phi = \phi(\mu), \tag{4.2}$$

$$F = \frac{L^4 e^{5A(\mu)}}{z^4} d\tau \wedge dx \wedge dy \wedge dz \wedge d\mu \tag{4.3}$$

$$+ \frac{L^4}{8} \cos \theta_1 \sin^3 \theta_1 \sin \theta_3 d\chi \wedge d\theta_1 \wedge d\theta_2 \wedge d\theta_3 \wedge d\theta_4. \tag{4.4}$$

We denote the time coordinate τ , because it differs from the time coordinate of our NRdCFT. We have chosen to represent the five-sphere as a Hopf fibration, as this allows for

simple implementation of the Null Melvin Twist in the next section. The explicit five-sphere metric in these coordinates is

$$ds_{S^5}^2 = ds_{\mathbb{P}^2}^2 + (d\chi + \mathcal{A})^2 \tag{4.5}$$

$$= d\chi^2 + \sin^2 \theta_1 d\chi d\theta_2 + \sin^2 \theta_1 \cos \theta_3 d\chi d\theta_4 + d\theta_1^2 + \frac{\sin^2 \theta_1}{4} d\theta_2^2 \tag{4.6}$$

$$+ \frac{\sin^2 \theta_1 \cos \theta_3}{2} d\theta_2 d\theta_4 + \frac{\sin^2 \theta_1}{4} d\theta_3^2 + \frac{\sin^2 \theta_1}{4} d\theta_4^2. \tag{4.7}$$

One can check that the above ansatz solves the full type IIB supergravity equations of motion, provided that the dilaton and warpfactor obey

$$\phi'(\mu) = ce^{-3A}, \tag{4.8}$$

$$A'(\mu) = \sqrt{e^{2A} - 1 + \frac{c^2}{24} e^{-6A}}. \tag{4.9}$$

Here, c is a constant that sets the strength of the jump in coupling across the defect in the dual field theory. The geometry is free from curvature singularities only if $c < \frac{9}{4\sqrt{2}}$.

4.2 The non-relativistic Janus solution

The Schrödinger geometry was successfully embedded into IIB supergravity in [14–16]. The papers [14, 16] accomplished this embedding through the use of the Null Melvin Twist (NMT). The NMT [19] is a series of boosts, T-dualities and twists that takes one solution to the IIB equations of motion and produces another. In particular, the NMT takes a geometry that is asymptotically AdS and transforms it into a geometry that is asymptotically Schrödinger. For this reason, it is a powerful tool to construct holographic duals to NRCFTs using preexisting supergravity solutions that are asymptotically AdS.

The details of the NMT are given in appendix A. Implementing the NMT requires the choice of two translational isometries. It is for this reason that we chose to write the spherical metric as a Hopf fibration in the previous subsection. Such a coordinate choice naturally identifies a U(1) isometry on the sphere, and we choose to perform our twist in this direction. Note that the choice of the U(1) fiber breaks the symmetry of the sphere down to U(1) × SU(3). This is reflected in the Melvinized Janus solution below.

Applying the NMT to the Janus solution is straightforward but tedious. The details of this calculation and the conventions we used are collected in appendix B. The resulting solution, in Einstein frame, is

$$ds_E^2 = L^2 e^{2A} \left(-\frac{e^{2A} e^\phi}{z^4} dt^2 + \frac{-2dt dv + dx^2 + dz^2}{z^2} + d\mu^2 \right) + L^2 ds_{S^5}^2 \tag{4.10}$$

$$\phi = \phi(\mu) \tag{4.11}$$

$$B = \frac{L^2 e^{2A} e^\phi}{z^2} (d\chi + \mathcal{A}) \wedge dt \tag{4.12}$$

$$F = \frac{L^4 e^{5A(\mu)}}{z^4} dt \wedge dv \wedge dx \wedge dz \wedge d\mu \tag{4.13}$$

$$+ \frac{L^4}{8} \cos \theta_1 \sin^3 \theta_1 \sin \theta_3 d\chi \wedge d\theta_1 \wedge d\theta_2 \wedge d\theta_3 \wedge d\theta_4. \tag{4.14}$$

Again, one can check that, as long as the derivatives of ϕ and A are still given by

$$\phi'(\mu) = ce^{-3A}, \tag{4.15}$$

$$A'(\mu) = \sqrt{e^{2A} - 1 + \frac{c^2}{24}e^{-6A}}, \tag{4.16}$$

this is a solution to the IIB equations of motion.

We see that (4.10) does indeed take the expected NRdCFT form, as described in (3.11).⁹ From the coefficient of dt^2 , we see that $B(\mu) = A(\mu) + \frac{\phi(\mu)}{2}$. Also, while the five-sphere has remained intact in the metric, the presence of the one-form $d\chi + \mathcal{A}$ breaks the spherical symmetry as described above.

We have thus constructed a particular example of the holographic NRdCFTs outlined in earlier sections of this paper. The dual NRdCFT is expected to consist of two regions of the twisted $\mathcal{N} = 4$ described in [14]. The two regions are separated by a codimension one defect, across which the coupling constant of the theories jumps. Unfortunately, no matter fields live on the defect, so the non-relativistic Janus solution is not a viable system to create p -wave superfluids as described in [4]. NRdCFTs with matter living on the defect could be constructed in a similar fashion by beginning with a gravity setup involving probe branes, but we do not study that case in this paper.

5 Conclusions and future directions

In this paper, we have begun an investigation into the holographic description of non-relativistic defect conformal field theories. We have found that the defect Schrödinger symmetry allows for the presence of an arbitrary function of the radial coordinate in the metric. This function does not affect the structure of the dOPE, which, in terms of defect operators, is the same as in the relativistic case. However, the arbitrary function does play a crucial role in the eigenvalue equation determining the dimensions of defect operators appearing in the dOPE for a ambient operator of fixed particle number. This feature helps elucidate how, in non-relativistic holography, a single bulk field is dual to a tower of boundary operators with differing particle number.

We have also managed to construct a particular example of an NRdCFT, which we have called the non-relativistic Janus solution. This solution was achieved by applying the Null Mevlin Twist to the relativistic Janus solution of type IIB supergravity. As discussed above, the non-relativistic Janus solution corresponds to a very particular choice of the arbitrary function allowed by the symmetries. The corresponding boundary theory has no matter localized to the defect. In this way, our specific example differs from the setups considered by Nishida in order to create p -wave superfluids.

The particularity of our solution naturally leads to the question of whether other NRdCFT solutions to IIB supergravity can be found. We will now present a general ansatz consistent with our symmetries and the accompanying equations of motion. The construction of a solution to these equations, other than the non-relativistic Janus solution, is left to future research.

⁹To have precise agreement with 3.11, one must enact a coordinate change from μ to r .

The most general metric consistent with the defect Schrödinger symmetry was written down earlier in (3.11). For a (2 + 1)-dimensional dual theory, the metric will read

$$ds^2 = e^{2A(r)} \left(-e^{2B(r)} \frac{dt^2}{z^4} + \frac{-2dvdt + dx^2 + dz^2}{z^2} \right) + dr^2 + (d\chi + \mathcal{A})^2 + ds_{\mathbb{CP}_2}^2. \quad (5.1)$$

The generators of the 1 + 1-dimensional defect NRCFT correspond to the following isometries of the metric:

$$H : \quad t = t' + a \quad (5.2)$$

$$P : \quad x = x' + b \quad (5.3)$$

$$N : \quad v = v' + c \quad (5.4)$$

$$M : \quad x = -x' \quad (5.5)$$

$$D : \quad t = \lambda^2 t', \quad v = v', \quad x = \lambda x', \quad z = \lambda z' \quad (5.6)$$

$$K : \quad x = x' + \beta t', \quad v = v' + \beta x' + \frac{1}{2} \beta^2 t' \quad (5.7)$$

$$C : \quad t = \frac{t'}{1 + \alpha t'}, \quad v = v' - \frac{\alpha((x')^2 + (z')^2)}{2(1 + \alpha t')}, \quad x = \frac{x'}{1 + \alpha t'}, \quad z = \frac{z'}{1 + \alpha z'}. \quad (5.8)$$

Note that there is also another discrete isometry of the metric,

$$T : t = -t', \quad v = -v'. \quad (5.9)$$

This corresponds to a time-reversal symmetry in the boundary theory. In the non-relativistic Janus solution, in order for this to be a symmetry transformation, we must simultaneously take $B_{(2)} = -B'_{(2)}$. Thus, we will require our Ansatz to have this symmetry. We also require that our solution preserves the $U(1) \times SO(4)$ isometry of the compact directions.

The H, P, N, D symmetries require the complex scalar field present in IIB supergravity (see [20] for our supergravity conventions) to depend only on the radial direction,

$$\tau = \tau(r). \quad (5.10)$$

Next, let us work out the possible terms in the two-form. First of all, the H, P and N transformations require the coefficients of all components to depend only on z and r . The M transformation prevents any two-form with an index in the x direction. The T transformation (remember that this includes $B \rightarrow -B$) requires every component to have an index in either the t or the v direction (but not both). Let us assume there is some component of B with an index in the v direction,

$$B_{\mu\nu} = f(r) d\mu \wedge dv. \quad (5.11)$$

Under the transformation K , dv will pick up a term that has a dx . The only other differential that would have a dx to cancel such a term is the x direction, but we have already established that B cannot have indices in the x direction. Thus, B also cannot have components in the v direction, and must therefore have one index in the t direction.

The remaining possibilities are

$$B = f_1(r, z)dt \wedge dz + f_2(r, z)dt \wedge dr + f_3(r, z)dt \wedge (d\chi + \mathcal{A}), \quad (5.12)$$

where the last term is the only possible term with indices in the compact directions that is consistent with the SO(4) isometry of \mathbb{CP}_2 . Finally, the D and C transformations uniquely determine the z dependence of the functions f_i . Our final most general two-form thus takes the form

$$B = \frac{e^{2C(r)}}{z^3}dt \wedge dz + \frac{e^{2D(r)}}{z^2}dt \wedge dr + \frac{e^{2E(r)}}{z^2}dt \wedge (d\chi + \mathcal{A}). \quad (5.13)$$

The remaining field in IIB supergravity is the self-dual five-form. In order to be consistent with the symmetries and to be self-dual, it must take the same form as in the non-relativistic Janus solution,

$$F = \frac{e^{5A(r)}}{z^4}dt \wedge dv \wedge dx \wedge dz \wedge dr \quad (5.14)$$

$$+ \frac{1}{8} \cos \theta_1 \sin^3 \theta_1 \sin \theta_3 d\chi \wedge d\theta_1 \wedge d\theta_2 \wedge d\theta_3 \wedge d\theta_4. \quad (5.15)$$

In order to simplify the search for a solution, one could demand that the axion vanish and that the two-form is real, as in the non-relativistic Janus solution. In the resulting equations of motion, $C(r)$ and $D(r)$ decouple from the other functions. The equations of motion from the Janus case carry over, with the addition of two more equations that must be satisfied by $B(r)$ and $E(r)$. The equations (independent of $C(r)$ and $D(r)$, which could be set to zero) reduce to the following set:

$$\phi'(\mu) = ce^{-3A}, \quad (5.16)$$

$$A'(\mu) = \sqrt{e^{2A} - 1 + \frac{c^2}{24}e^{-6A}} \quad (5.17)$$

$$0 = 8e^{2A} - 6 + 2A'\phi' - 2E'\phi' - 2A'E' - 4(E')^2 - 2E'' \quad (5.18)$$

$$0 = e^{2A+2B} (10 + 6A'B' + 4(B')^2 + 4B'\phi' + c^2e^{-6A} + 2B'') - e^{4E} (4 + 8e^{2A} + 4(E')^2 + 4E'\phi' + c^2e^{-6A}). \quad (5.19)$$

We have not been able to find a simple solution to these equations.

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A The Null Melvin Twist

The Null Melvin Twist, first introduced in [19], is a solution-generating technique that takes a solution to supergravity as an input and produces a second solution to supergravity, generally containing some lightlike NS-NS flux. The procedure involves six steps. They are as follows:

1. Boost by γ along a translationally invariant direction y .

2. T-dualize along the y -direction.
3. Choose a one-form (for us it will be $d\chi + \mathcal{A}$) and shift it by a constant amount α in the y direction.
4. T-dualize back along the y direction.
5. Boost back by $-\gamma$ in the y direction.
6. Take the limit $\gamma \rightarrow \infty$, $\alpha \rightarrow 0$ such that $\beta = \frac{1}{2}\alpha e^\gamma$ is held fixed.

In order to enact the T-dualities in steps two and four, we will use the Buscher rules [21], which apply to the fields of IIB supergravity in the string frame. Under these rules, the fields in the NS-NS sector transform into new, primed fields as

$$g'_{yy} = \frac{1}{g_{yy}}, \quad g'_{ay} = \frac{B_{ay}}{g_{yy}}, \quad g'_{ab} = g_{ab} - \frac{g_{ay}g_{yb} + B_{ay}B_{yb}}{g_{yy}}, \quad (\text{A.1})$$

$$B'_{ay} = \frac{g_{ay}}{g_{yy}}, \quad B'_{ab} = B_{ab} - \frac{g_{ay}B_{yb} + B_{ay}g_{yb}}{g_{yy}}, \quad (\text{A.2})$$

$$\phi' = \phi - \frac{1}{2} \log g_{yy}. \quad (\text{A.3})$$

Here, a and b are any direction other than y .

B The NMT of Janus

In this appendix we explicitly implement the NMT on the Janus solution. Our discussion closely follows [16].

In order to use the Buscher rules as listed in the previous section, we must start with a solution in string frame. Before carrying out the NMT, we must therefore multiply the Janus metric in section 4.1 by $e^{\phi/2}$.

The five-form is unaffected by the NMT. The argument is exactly the same as in [16]. Thus, we will only be concerned with the dilaton, the B field, and the following components of the string frame metric:

$$ds^2 = -\frac{L^2 e^{2A} e^{\phi/2}}{z^2} d\tau^2 + \frac{L^2 e^{2A} e^{\phi/2}}{z^2} dy^2 + L^2 e^{\phi/2} (d\chi + \mathcal{A})^2. \quad (\text{B.1})$$

All other components of the metric are unaffected by the NMT. We will denote the original dilaton present in the Janus solution as ϕ_0 .

1. The first step of the NMT is to boost by γ in the y direction,

$$\tau \rightarrow \cosh \gamma \tau - \sinh \gamma y, \quad y \rightarrow \cosh \gamma y - \sinh \gamma \tau. \quad (\text{B.2})$$

This has no effect on our fields, as our metric is manifestly boost invariant and the initial B field is zero.

2. The second step is to T-dualize in the y direction. Using the Buscher rules described in the previous section, the nontrivial transformations are:

$$g_{yy} \rightarrow \frac{1}{g_{yy}} = \frac{z^2}{L^2 e^{2A} e^{\phi/2}} \quad (\text{B.3})$$

$$\phi \rightarrow \phi - \frac{1}{2} \log g_{yy} = \phi_0 - \frac{1}{2} \log \frac{L^2 e^{2A} e^{\phi/2}}{z^2}. \quad (\text{B.4})$$

After T-dualizing, our metric dilaton and B-field are

$$ds^2 = -\frac{L^2 e^{2A} e^{\phi/2}}{z^2} d\tau^2 + \frac{z^2}{L^2 e^{2A} e^{\phi/2}} dy^2 + L^2 e^{\phi/2} (d\chi + \mathcal{A})^2 \quad (\text{B.5})$$

$$\phi = \phi_0 - \frac{1}{2} \log \frac{L^2 e^{2A} e^{\phi/2}}{z^2} \quad (\text{B.6})$$

$$B = 0. \quad (\text{B.7})$$

3. The third step is to shift $\chi \rightarrow \chi + \alpha y$, where α is a constant. Only the metric is affected by this procedure. Our fields are now

$$ds^2 = -\frac{L^2 e^{2A} e^{\phi/2}}{z^2} d\tau^2 + \frac{z^2 + L^4 e^{2A} e^{\phi} \alpha^2}{L^2 e^{2A} e^{\phi/2}} dy^2 + 2\alpha L^2 e^{\phi/2} dy (d\chi + \mathcal{A}) + L^2 e^{\phi/2} (d\chi + \mathcal{A})^2 \quad (\text{B.8})$$

$$\phi = \phi_0 - \frac{1}{2} \log \frac{L^2 e^{2A} e^{\phi/2}}{z^2} \quad (\text{B.9})$$

$$B = 0. \quad (\text{B.10})$$

4. The next step is to T-dualize back along the y direction. We use the shorthand χ to represent indices along the direction $(d\chi + \mathcal{A})$. Using the Buscher rules again, we find

$$g_{yy} \rightarrow \frac{L^2 e^{2A} e^{\phi/2}}{z^2 + L^4 e^{2A} e^{\phi} \alpha^2} \quad (\text{B.11})$$

$$g_{\chi y} \rightarrow \frac{B_{\chi y}}{g_{yy}} = 0 \quad (\text{B.12})$$

$$g_{\chi\chi} \rightarrow g_{\chi\chi} - \frac{g_{\chi y}^2}{g_{yy}} = \frac{L^2 z^2 e^{\phi/2}}{z^2 + L^4 e^{2A} e^{\phi} \alpha^2} \quad (\text{B.13})$$

$$\phi \rightarrow \phi_0 - \frac{1}{2} \log \frac{z^2 + L^4 e^{2A} e^{\phi} \alpha^2}{z^2} \quad (\text{B.14})$$

$$B_{\chi y} \rightarrow \frac{g_{\chi y}}{g_{yy}} = \frac{\alpha L^4 e^{2A} e^{\phi}}{z^2 + L^4 e^{2A} e^{\phi} \alpha^2}. \quad (\text{B.15})$$

After this, our fields read

$$ds^2 = -\frac{L^2 e^{2A} e^{\phi/2}}{z^2} d\tau^2 + \frac{L^2 e^{2A} e^{\phi/2}}{z^2 + L^4 e^{2A} e^{\phi} \alpha^2} dy^2 + \frac{L^2 z^2 e^{\phi/2}}{z^2 + L^4 e^{2A} e^{\phi} \alpha^2} (d\chi + \mathcal{A})^2 \quad (\text{B.16})$$

$$\phi = \phi_0 - \frac{1}{2} \log \frac{z^2 + L^4 e^{2A} e^{\phi} \alpha^2}{z^2} \quad (\text{B.17})$$

$$B = \frac{\alpha L^4 e^{2A} e^{\phi}}{z^2 + L^4 e^{2A} e^{\phi} \alpha^2} (d\chi + \mathcal{A}) \wedge dy. \quad (\text{B.18})$$

5. The fifth step is to boost back by $-\gamma$ in the y direction,

$$\tau \rightarrow \cosh \gamma \tau + \sinh \gamma y, \quad y \rightarrow \cosh \gamma y + \sinh \gamma \tau. \quad (\text{B.19})$$

After this boost, our fields are

$$ds^2 = -\frac{L^2 e^{2A} e^{\phi/2}}{z^2} \frac{z^2 + L^4 e^{2A} e^\phi \alpha^2 \cosh^2 \gamma}{z^2 + L^4 e^{2A} e^\phi \alpha^2} d\tau^2 - \frac{2L^6 e^{4A} e^{3\phi/2} \alpha^2 \cosh \gamma \sinh \gamma}{z^2 (z^2 + L^4 e^{2A} e^\phi \alpha^2)} dy d\tau \\ + \frac{L^2 e^{2A} e^{\phi/2}}{z^2} \frac{z^2 - L^4 e^{2A} e^\phi \alpha^2 \sinh^2 \gamma}{z^2 + L^4 e^{2A} e^\phi \alpha^2} dy^2 + \frac{L^2 z^2 e^{\phi/2}}{z^2 + L^4 e^{2A} e^\phi \alpha^2} (d\chi + \mathcal{A})^2 \quad (\text{B.20})$$

$$\phi = \phi_0 - \frac{1}{2} \log \frac{z^2 + L^4 e^{2A} e^\phi \alpha^2}{z^2} \quad (\text{B.21})$$

$$B = \frac{\alpha L^4 e^{2A} e^\phi}{z^2 + L^4 e^{2A} e^\phi \alpha^2} (d\chi + \mathcal{A}) \wedge (\cosh \gamma dy + \sinh \gamma d\tau). \quad (\text{B.22})$$

6. The final step is to take the limit $\alpha \rightarrow 0, \gamma \rightarrow \infty$ such that $\frac{e^\gamma \alpha}{2} = \beta$ is held fixed. Functionally, this means wherever we have a $\sinh \gamma$ or $\cosh \gamma$ times α we can replace it by β , and wherever we just have an α we can drop those terms.

After taking the limit, our fields become

$$ds^2 = -\frac{L^2 e^{2A} e^{\phi/2} (z^2 + L^4 e^{2A} e^\phi \beta^2)}{z^4} d\tau^2 - \frac{2L^6 e^{4A} e^{3\phi/2} \beta^2}{z^4} dy d\tau \\ + \frac{L^2 e^{2A} e^{\phi/2} (z^2 - L^4 e^{2A} e^\phi \beta^2)}{z^4} dy^2 + L^2 e^{\phi/2} (d\chi + \mathcal{A})^2 \quad (\text{B.23})$$

$$\phi = \phi_0 \quad (\text{B.24})$$

$$B = \frac{\beta L^4 e^{2A} e^\phi}{z^2} (d\chi + \mathcal{A}) \wedge (dy + d\tau). \quad (\text{B.25})$$

This completes the NMT procedure. However, to make the Schrödinger symmetry manifest, we will change to lightcone coordinates,

$$\tau = \frac{1}{\sqrt{2}} (t + v), \quad y = \frac{1}{\sqrt{2}} (t - v). \quad (\text{B.26})$$

In these coordinates, our fields are

$$ds^2 = -\frac{2\beta^2 L^6 e^{4A} e^{3\phi/2}}{z^4} dt^2 - \frac{2L^2 e^{2A} e^{\phi/2}}{z^2} dt dv + L^2 e^{\phi/2} (d\chi + \mathcal{A})^2 \quad (\text{B.27})$$

$$\phi = \phi_0 \quad (\text{B.28})$$

$$B = \frac{\sqrt{2} \beta L^4 e^{2A} e^\phi}{z^2} (d\chi + \mathcal{A}) \wedge (dt). \quad (\text{B.29})$$

Finally, to clean things up, we make the change of coordinates

$$t \rightarrow \frac{t}{\sqrt{2}\beta L^2} \quad v \rightarrow \sqrt{2}\beta L^2 v, \quad (\text{B.30})$$

leaving

$$ds^2 = L^2 e^{2A} e^{\phi/2} \left(-\frac{e^{2A} e^{\phi}}{z^4} dt^2 + \frac{-2dt dv}{z^2} \right) + L^2 e^{\phi/2} (d\chi + \mathcal{A})^2 \quad (\text{B.31})$$

$$\phi = \phi_0 \quad (\text{B.32})$$

$$B = \frac{L^2 e^{2A} e^{\phi}}{z^2} (d\chi + \mathcal{A}) \wedge (dt). \quad (\text{B.33})$$

Reverting back to Einstein frame, and restoring the unaffected components of the metric, we recover the non-relativistic Janus solution written down in 4.2.

References

- [1] W. Ketterle and M.W. Zwierlein, *Making, probing and understanding ultracold fermi gases*, [arXiv:0801.2500](#).
- [2] S. Giorgini, L.P. Pitaevskii and S. Stringari, *Theory of ultracold atomic Fermi gases*, *Rev. Mod. Phys.* **80** (2008) 1215 [[SPIRES](#)].
- [3] I. Bloch, J. Dalibard and W. Zwerger, *Many-body physics with ultracold gases*, *Rev. Mod. Phys.* **80** (2008) 885 [[SPIRES](#)].
- [4] Y. Nishida, *Induced p-wave superfluidity in two dimensions: brane world in cold atoms and nonrelativistic defect CFTs*, *Annals Phys.* **324** (2009) 897 [[arXiv:0810.1321](#)] [[SPIRES](#)].
- [5] D.T. Son, *Toward an AdS/cold atoms correspondence: a geometric realization of the Schrödinger symmetry*, *Phys. Rev. D* **78** (2008) 046003 [[arXiv:0804.3972](#)] [[SPIRES](#)].
- [6] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, *Phys. Rev. Lett.* **101** (2008) 061601 [[arXiv:0804.4053](#)] [[SPIRES](#)].
- [7] O. Aharony, O. DeWolfe, D.Z. Freedman and A. Karch, *Defect conformal field theory and locally localized gravity*, *JHEP* **07** (2003) 030 [[hep-th/0303249](#)] [[SPIRES](#)].
- [8] D. Bak, M. Gutperle and S. Hirano, *A dilatonic deformation of AdS₅ and its field theory dual*, *JHEP* **05** (2003) 072 [[hep-th/0304129](#)] [[SPIRES](#)].
- [9] D.M. McAvity and H. Osborn, *Conformal field theories near a boundary in general dimensions*, *Nucl. Phys. B* **455** (1995) 522 [[cond-mat/9505127](#)] [[SPIRES](#)].
- [10] H. Osborn and A.C. Petkou, *Implications of conformal invariance in field theories for general dimensions*, *Ann. Phys.* **231** (1994) 311 [[hep-th/9307010](#)] [[SPIRES](#)].
- [11] D.M. Hofman and J. Maldacena, *Conformal collider physics: energy and charge correlations*, *JHEP* **05** (2008) 012 [[arXiv:0803.1467](#)] [[SPIRES](#)].
- [12] M. Henkel, *Schrödinger invariance in strongly anisotropic critical systems*, *J. Stat. Phys.* **75** (1994) 1023 [[hep-th/9310081](#)] [[SPIRES](#)].
- [13] Y. Nishida and D.T. Son, *Nonrelativistic conformal field theories*, *Phys. Rev. D* **76** (2007) 086004 [[arXiv:0706.3746](#)] [[SPIRES](#)].
- [14] C.P. Herzog, M. Rangamani and S.F. Ross, *Heating up Galilean holography*, *JHEP* **11** (2008) 080 [[arXiv:0807.1099](#)] [[SPIRES](#)].
- [15] J. Maldacena, D. Martelli and Y. Tachikawa, *Comments on string theory backgrounds with non-relativistic conformal symmetry*, *JHEP* **10** (2008) 072 [[arXiv:0807.1100](#)] [[SPIRES](#)].

- [16] A. Adams, K. Balasubramanian and J. McGreevy, *Hot spacetimes for cold atoms*, *JHEP* **11** (2008) 059 [[arXiv:0807.1111](#)] [[SPIRES](#)].
- [17] D.Z. Freedman, C. Núñez, M. Schnabl and K. Skenderis, *Fake supergravity and domain wall stability*, *Phys. Rev. D* **69** (2004) 104027 [[hep-th/0312055](#)] [[SPIRES](#)].
- [18] A.B. Clark, D.Z. Freedman, A. Karch and M. Schnabl, *The dual of Janus ($(<:) \leftrightarrow (:>)$) an interface CFT*, *Phys. Rev. D* **71** (2005) 066003 [[hep-th/0407073](#)] [[SPIRES](#)].
- [19] E.G. Gimon, A. Hashimoto, V.E. Hubeny, O. Lunin and M. Rangamani, *Black strings in asymptotically plane wave geometries*, *JHEP* **08** (2003) 035 [[hep-th/0306131](#)] [[SPIRES](#)].
- [20] J.H. Schwarz, *Covariant Field Equations of chiral $N = 2$ $D = 10$ supergravity*, *Nucl. Phys. B* **226** (1983) 269 [[SPIRES](#)].
- [21] T.H. Buscher, *A symmetry of the string background field equations*, *Phys. Lett. B* **194** (1987) 59 [[SPIRES](#)].